Identification of Linear Systems by Poisson Moment Functionals in the Presence of Noise

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Abstract

THE parameters of several increasingly complex linear systems are identified in the presence of additive noise, using the method of Poisson Moment Functionals (PMF). Two formulations are examined. The first identifies the first order state equations of the structure from measurements of generalized displacements, velocities, and forces. The second identifies the second order equations of motion from measurements of the generalized displacements and forces. The initial conditions of the system are arbitrary and are identified. Comparisons are made with a second-order auto regressive moving average (ARMA) method.

Contents

The efficacy of linear system identification by PMF has been summarized elsewhere.[1] When applied to the identification of structural systems,[2] the method is especially attractive as it results in direct identification of the state equations or of the second order equations of motion; either of which can be used for additional analysis.[3] Shown herein are results of the extension of previous work to include arbitrary initial conditions; a new algorithm for the case where only generalized displacements are observed; and a comparison with a second order ARMA method discussed elsewhere in detail.[4] In the following, we will discuss only results of the "multiple time" formulation, recognizing that a "multiple moment" formulation is also possible.[2] We present results of the fully determined formulation, recognizing that overdetermination may be required to average out measurement noise. Finally, we have assumed that the order of the system is known a priori since we are concerned primarily with the relative performance of the algorithms in presence of zero mean additive noise. The important issue of system order determination will be addressed in subsequent work, presently in progress.

The PMF transform operates on a process signal $f(t)$ over the interval $[0,t_0]$ and converts it to a set of real numbers

$$f_k^{t_0} = f_k(t_0) = \int_0^{t_0} f(t) p_k(t_0 - t)\,\mathrm{d}t, \qquad k = 0,1,2,\ldots \tag{1}$$

$$p_k^t = p_k(t) = t^k e^{-\lambda t}/k! \tag{2}$$

$$\lambda(\text{real}) > 0 \tag{3a}$$

$$f_{-1}(t_0) = f(t_0) \tag{3b}$$

Let $f(t)$ and $\mathrm{d}f(t)/\mathrm{d}t$ be arbitrary at $t=0$, and let $f_k^{(1)}(t_0)$ denote the kth PMF of $\mathrm{d}f(t)/\mathrm{d}t$. Then[1]

$$f_k^{(1)}(t_0) = -\lambda f_k(t_0) + f_{k-1}(t_0) - f(0) p_k(t_0), \quad k = 0,1,2,\ldots \tag{4}$$

Application of Eqs. (1-4) to the identification of the n dimensional equations of state, when all of the states are observable, has been discussed previously for the case of quiescent initial conditions.[2] Briefly, the matrix $[C]$, containing the system and input matrices and vector of initial conditions, is identified from

$$[C] = [Y][\hat{\Psi}][\hat{\Phi}]^{-1} \tag{5}$$

where the matrix $[\hat{\Phi}]$ has been modified to include effects of arbitrary initial conditions.

Application of Eqs. (1-4) to the identification of second order systems, having $n/2$ degrees of freedom, when only the generalized displacements are observable, again leads to a system defined by Eq. (5). Now, however, the matrix $[C]$ contains the stiffness, damping and input matrices, premultiplied by the inverse of the mass matrix, as well as the initial displacement and initial velocity vectors.

Two systems were identified using each of the three algorithms. The first, a single degree of freedom system, is given by

$$\bar{c} = 1, \quad \bar{k} = 10, \quad \bar{g} = 5, \quad x(0) = 1, \quad \dot{x}(0) = 2$$

and the second, a three degree of freedom system, is given by

$$[\bar{c}] = 0.10[I], \quad [\bar{k}] = \begin{bmatrix} 96 & -48 & 0 \\ -48 & 96 & -48 \\ 0 & -48 & 48 \end{bmatrix}$$

$$\boldsymbol{g} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \boldsymbol{x}(0) = \boldsymbol{0}, \quad \dot{\boldsymbol{x}}(0) = \boldsymbol{1}$$

The PMF algorithms were implemented on the CDC Cyber 175 at the University of Illinois, Urbana-Champaign. The equations of motion were solved analytically for $u(t) = \sin(4\pi t)$. Note that, for the single input case, there are no restrictions on $u(t)$; however, for the multiple input case, proportional inputs result in a noninvertible $[\hat{\Phi}]$ matrix. The peak response of each state was determined, and zero mean Gaussian noise was added as a percentage of peak. The same was done for the input. Noise was generated using IMSL routine GGUBFS.[6] For the fully observable formulation, PMF's were computed at 4 and 8 equally spaced times over 1 s for the 1 and 3 DOF systems, respectively. The partially observable formulation required 5 and 9 times, respectively. Numerical integration was performed using IMSL routine DCADRE to simulate the analog Poisson filter chain. Fi-

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Table 1 Identification error for 1 s of data in the presence of zero mean additive noise. PMF1 is fully observable; PMF2, partially observable; ARMA1, 25 records; ARMA2, 100 records

		Relative identification error			
System	% Noise	PMF1	PMF2	ARMA1	ARMA2
1 DOF	0	7.49 E-7	7.00 E-7	8.69 E-13	2.49 E-10
	10	7.49 E-7	6.91 E-7	2.96 E-3	1.23 E-2
	50	7.49 E-7	6.91 E-7	1.47 E-2	5.97 E-2
	100	7.49 E-7	6.90 E-7	2.91 E-2	1.15 E-1
3 DOF	0	1.89 E-5	1.79 E-4	5.98 E-8	2.72 E-7
	10	2.11 E-5	1.80 E-4	1.08 E-1	4.69 E-1
	50	2.10 E-5	1.80 E-4	4.34 E-1	8.86 E-1
	100	1.88 E-5	1.80 E-4	6.53 E-1	9.52 E-1

nally, the identified $[C]$ matrix was determined using IMSL routine OFIMAS. Multiple values of additive noise were examined. In all cases, $k=\lambda=1$.

The ARMA algorithm was also implemented on the Cyber 175. Data were generated at equal time intervals over 1 s. For both systems, time increments of .04 and .01 s and multiple noise levels added to the responses were investigated. IMSL routine OFIMAS was used to solve the ordinary least square problem for the identified matrix $[C]$.

For both PMF and ARMA methods, the relative identification error was defined by the ratio of Frobenius norms,[5]

$$\text{Relative error} = ||[C]_{\text{exact}} - [C]_{\text{identified}}||/||[C]_{\text{exact}}|| \quad (6)$$

Results of the numerical experiments are summarized in Table 1, where the relative error indicates the order of magnitude of the error in each matrix entry.

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